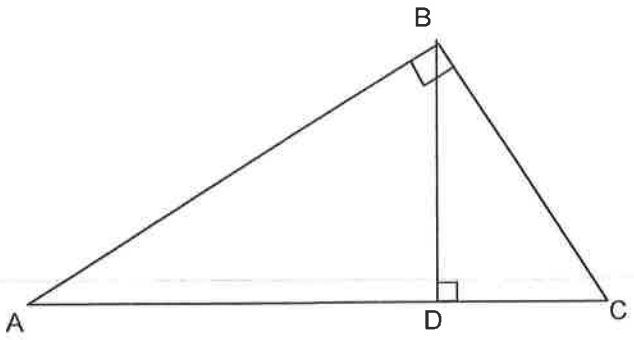
Honors Geometry Name Key

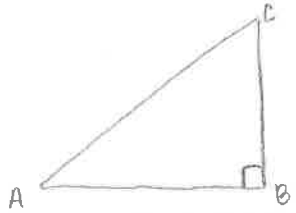
Section 7.4 Notes

Similarity in Right Triangles (Geometric Mean)

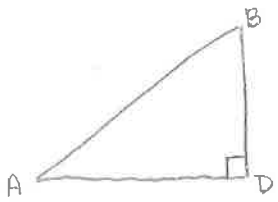
Starter Problem: In the picture below are 3 similar triangles: the large triangle, and the small triangles to the left and right. Draw the 3 triangles separately, so that corresponding parts are in the same position. Then, complete the proportions.



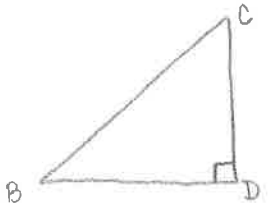
△ABC (big triangle)



△ABD (left triangle)



△CBD (right triangle)



Proportions:

#1 Comparing left and right triangles:	#2 Comparing big triangle to left triangle:	#3 Comparing big triangle to right triangle:
$\frac{AD}{BD} = \frac{BD}{CD} = \frac{AB}{BC}$	$\frac{AC}{AB} = \frac{AB}{AD} = \frac{CB}{BD}$	$\frac{AC}{BC} = \frac{BC}{DC} = \frac{AB}{BD}$

Geometric Mean:

The geometric mean of two numbers is the number x that satisifies $\frac{a}{x} = \frac{x}{b}$, so $ab = x^2$

In the figure above, the following are geometric means:

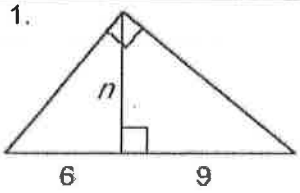
- BD (the altitude) is the geometric mean between AD and CD
- AB (left leg) is the geometric mean between AC and AD
- BC (right leg) is the geometric mean between AC and DC

Theorems

Altitude Geometric Mean (Heartbeat): In a right triangle, the length of the altitude from the right angle is the geometric mean between the two segments formed on the hypotenuse.
Leg Geometric Mean (Boomerang): In a right triangle, with an altitude drawn from right angle, the length of each leg is the geometric mean between the whole hypotenuse and the segment of the hypotenuse that is adjacent to the given leg.

Practice: Find the exact value (unless otherwise stated) of the given variable(s):

1.



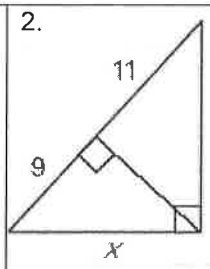
$\frac{6}{n} = \frac{n}{9}$ (heartbeat)

$n^2 = 54$

$n = \sqrt{54}$ $\quad n = 3\sqrt{6}$

$\sqrt{9}\sqrt{6}$

2.



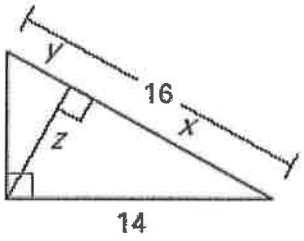
$\frac{20}{x} = \frac{x}{9}$ (boomerang)

$x^2 = 180$

$x = \sqrt{180}$ $\quad x = 6\sqrt{5}$

$\sqrt{36}\sqrt{5}$

3. Round to the nearest tenth



$\frac{16}{14} = \frac{14}{x}$

$16x = 196$

$x = 12.3$

$16 - 12.3 = 3.7$

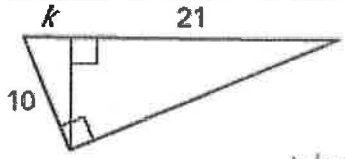
$y = 3.7$

$\frac{12.3}{z} = \frac{z}{3.7}$

$z^2 = 45.51$

$z = 6.7$

4.



$\frac{21+k}{10} = \frac{10}{k}$

$k(21+k) = 100$

$k^2 + 21k = 100$

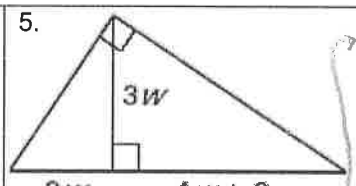
$k^2 + 21k - 100 = 0$

$(k-4)(k+25) = 0$

$k-4=0 \quad k+25=0$

$k = 4$ $\quad k \neq -25$

5.



$\frac{2w}{3w} = \frac{3w}{4w+3}$

$2w(4w+3) = 9w^2$

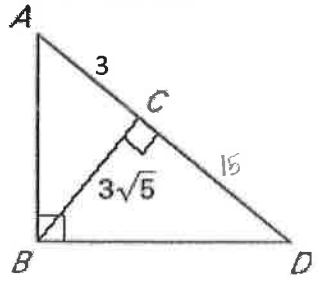
$8w^2 + 6w = 9w^2$

$w^2 - 6w = 0$

$w(w-6) = 0$

$w \neq 0$ $\quad w = 6$

6. Find CD and BD:



$\frac{3}{3\sqrt{5}} = \frac{3\sqrt{5}}{CD}$

$3CD = 45$

$CD = 15$

$\frac{18}{BD} = \frac{BD}{15}$

$BD^2 = 270$

$BD = \sqrt{270}$

$\sqrt{9}\sqrt{30}$

$BD = 3\sqrt{30}$